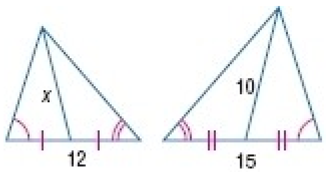
Find *x*.

1. 

SOLUTION:

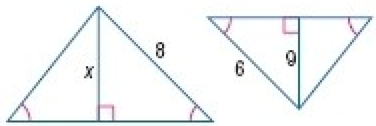
By AA Similarity, the given two triangles are similar. Additionally, we see the segments marked *x* and 10 are medians because they intersect the opposite side at its midpoint.

Theorem 7.10 states that if two triangles are similar, the lengths of corresponding medians are proportional to the lengths of corresponding sides. Therefore,

$$\frac{x}{10} = \frac{12}{15}$$
$$15x = 120$$
$$x = 8$$

ANSWER:

8

2. 

SOLUTION:

By AA Similarity, the given two triangles are similar. Additionally, we see the segments marked *x* and 10 are medians because they intersect the opposite side at its midpoint.

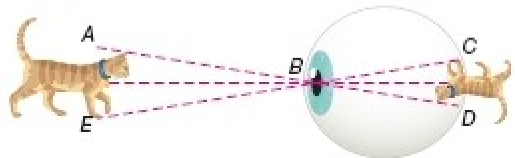
Theorem 7.8 states that if two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides. Therefore,

$$\frac{x}{9} = \frac{8}{6}$$
$$6x = 72$$
$$x = 12$$

ANSWER:

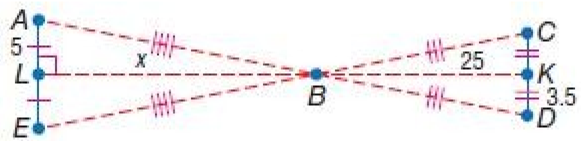
12

3. **VISION** A cat that is 10 inches tall forms a retinal image that is 7 millimeters tall. If $\Delta ABE \sim \Delta DBC$ and the distance from the pupil to the retina is 25 millimeters, how far away from your pupil is the cat?



SOLUTION:

If two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides. Let the altitude from B meet AE at L and CD at K.



Since triangle ABE and DBC are similar, triangle ALB and triangle DKB are similar.

We know that BK = 25 mm, DK = 3.5 mm and AL = 5 inches. Use the Proportionality Theorem.

$$\frac{BK}{KD} = \frac{LB}{AL}$$

Substitute in given values and solve for LB.

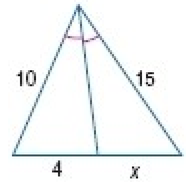
$$\frac{25}{3.5} = \frac{LB}{5}$$
$$3.5LB = 12.5$$
$$LB \approx 35.7$$

The cat is 35.7 ft away from your pupil.

ANSWER:

35.7 ft

Find the value of each variable.

4. 

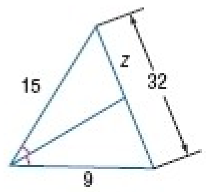
SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{x}{4} = \frac{15}{10}$$
$$10x = 60$$
$$x = 6$$

ANSWER:

6

5. 

SOLUTION:

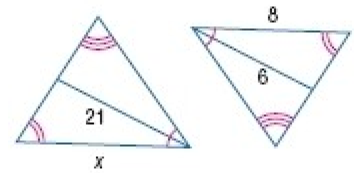
An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{z}{32-z}=\frac{15}{9}$$
$$9z=480-15z$$
$$24z=480$$
$$z=20$$

ANSWER:

20

Find *x*.

6. 

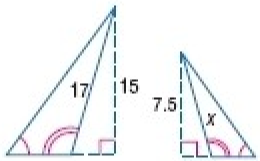
SOLUTION:

By AA Similarity, the given two triangles are similar. Theorem 7.9 states that if two triangles are similar, the lengths of corresponding angle bisectors are proportional to the lengths of corresponding sides. Therefore,

$$\frac{x}{8}=\frac{21}{6}$$
$$6x=168$$
$$x=28$$

ANSWER:

28

7. 

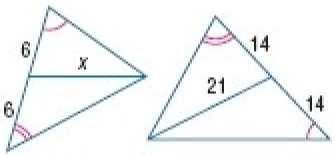
SOLUTION:

By AA Similarity, the given two triangles are similar. Theorem 7.8 states that if two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides.we know that the sides marked 15 and 7.5 are altitudes because they are perpendicular to the side opposite a vertex of the triangle. Therefore,

$$\frac{x}{17} = \frac{7.5}{15}$$
$$15x = 127.5$$
$$x = 8.5$$

ANSWER:

8.5

8. 

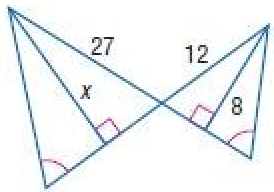
SOLUTION:

By AA Similarity, the given two triangles are similar. Theorem 7.10 states that if two triangles are similar, the lengths of corresponding medians are proportional to the lengths of corresponding sides. We know that the segments marked x and 21 are medians because they intersect the opposite side at its midpoint. Therefore,

$$\frac{x}{21} = \frac{12}{28}$$
$$28x = 252$$
$$x = 9$$

ANSWER:

9

9. 

SOLUTION:

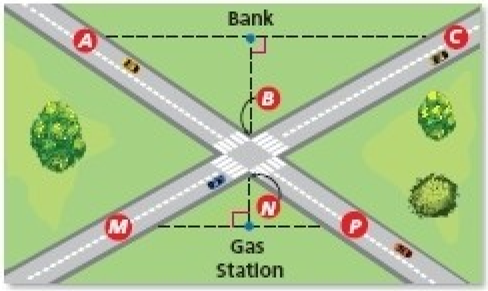
By AA Similarity, the given two triangles are similar.We know that the segments labeled x and 8 are altitudes because they are perpendicular to the side opposite a vertex. theorem 7.8 states that if two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides. Therefore,

$$\frac{x}{8} = \frac{27}{12}$$
$$12x = 216$$
$$x = 18$$

ANSWER:

18

10. **ROADWAYS** The intersection of the two roads shown forms two similar triangles. If AC is 382 feet, MP is 248 feet, and the gas station is 50 feet from the intersection, how far from the intersection is the bank?



SOLUTION:

If two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides. Let *x* be the distance between the intersection and the bank.

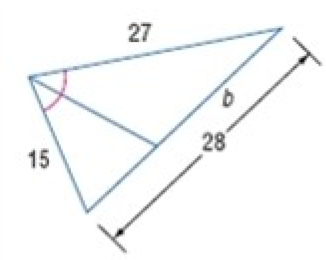
$$\frac{\text{The distance from the Bank to point B}}{\text{The distance from the Gas Station to point B}} = \frac{AC}{MP}$$
$$\frac{x}{50} = \frac{382}{248}$$
$$248x = 19100$$
$$x \approx 77$$

Therefore, the distance between the intersection at point B and the bank is about 77 feet.

ANSWER:

about 77 ft

Find the value of each variable. Note that figures are not drawn to scale.

11. 

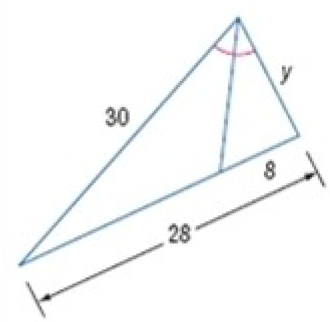
SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{15}{27}=\frac{28-b}{b}$$
$$15b=27(28-b)$$
$$15b=756-27b$$
$$42b=756$$
$$b=18$$

ANSWER:

18

12. 

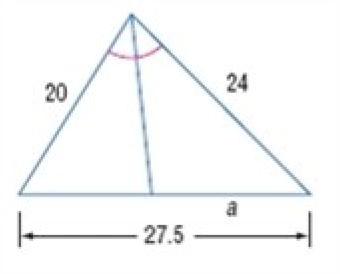
SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{8}{28-8}=\frac{y}{30}$$
$$\frac{8}{20}=\frac{y}{30}$$
$$20y=240$$
$$y=12$$

ANSWER:

12

13. 

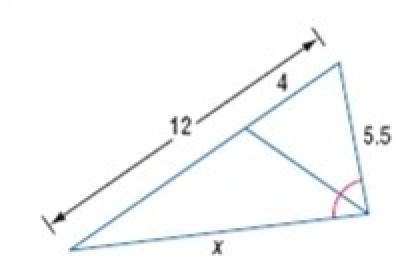
SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{20}{24}=\frac{27.5-a}{a}$$
$$20a=24(27.5-a)$$
$$20a=660-24a$$
$$44a=660$$
$$a=15$$

ANSWER:

15

14. 

SOLUTION:

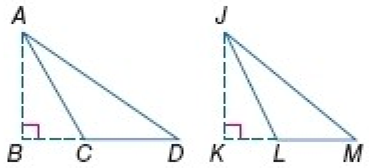
An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{x}{5.5}=\frac{12-4}{4}$$
$$\frac{x}{5.5}=\frac{8}{4}$$
$$4x=44$$
$$x=11$$

ANSWER:

11

15. **ALGEBRA** If $\overline{AB}$ and $\overline{JK}$ are altitudes, $\Delta DAC \sim \Delta MJL$, $AB = 9$, $AD = 4x - 8$, $JK = 21$, and $JM = 5x + 3$, find x.



SOLUTION:

Form a proportion.

$$\frac{AB}{JK} = \frac{AD}{JM}$$

Substitute.

$$\frac{9}{21} = \frac{4x-8}{5x+3}$$

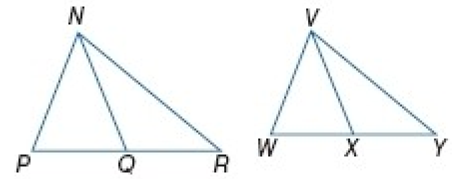
Solve for x.

$$\begin{aligned} 9(5x+3) &= 21(4x-8) \\ 45x + 27 &= 84x - 168 \\ 45x - 84x &= -27 - 168 \\ -39x &= -195 \\ x &= 5 \end{aligned}$$

ANSWER:

5

16. **ALGEBRA** If $\overline{NQ}$ and $\overline{VX}$ are medians, $\Delta PNR \sim \Delta WVY$, $NQ = 8$, $PR = 12$, $WY = 7x - 1$, and $VX = 4x + 2$, find x.



SOLUTION:

If two triangles are similar, the lengths of corresponding medians are proportional to the lengths of corresponding sides.

$$\frac{NQ}{VX} = \frac{PR}{WY}$$

Substitute.

$$\frac{8}{4x+2} = \frac{12}{7x-1}$$

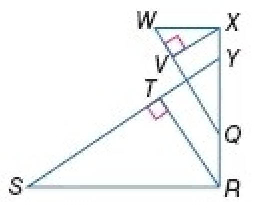
Solve for x.

$$\begin{aligned} 8(7x-1) &= 12(4x+2) \\ 56x - 8 &= 48x + 24 \\ 8x &= 32 \\ x &= 4 \end{aligned}$$

ANSWER:

4

17. If $\Delta SRY \sim \Delta WXQ, \overline{RT}$ is an altitude of $\Delta SRY, \overline{XV}$ is an altitude of ΔWXQ, RT = 5, RQ = 4, QY = 6, and YX = 2, find XV.



SOLUTION:

If two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides.

$$\frac{TR}{VX} = \frac{YR}{XQ}$$

$$\frac{TR}{VX} = \frac{RQ + QY}{XY + YQ}$$

Substitute.

$$\frac{5}{VX} = \frac{10}{8}$$

$$10VX = 40$$

$$VX = 4$$

ANSWER:

4

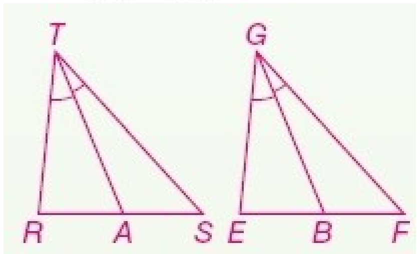
18. **PROOF** Write a paragraph proof of Theorem 7.9.

SOLUTION:

Theorem 7.9 states that if two triangles are similar, the lengths of corresponding angle bisectors are proportional to the lengths of corresponding sides. In your proof, since you are given that two triangles are similar, you can use congruent corresponding angle measures to prove that $\angle R \cong \angle E$. Then, since $\angle T$ and $\angle G$ are congruent and bisected, their their halves are also congruent. Therefore, by AA Similarity theorem, $\Delta RTA \sim \Delta EGB$ and, consequently, corresponding sides are proportional to each other.

Given: $\Delta RTS \sim \Delta EGF, \overline{TA}$ and $\overline{GB}$ are angle bisectors.

Prove: $\frac{TA}{GB} = \frac{RT}{EG}$

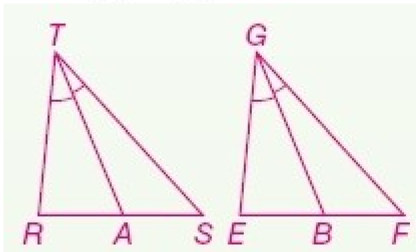


Proof: Because corresponding angles of similar triangles are congruent, $\angle R \cong \angle E$ and $\angle RTS \cong \angle EGF$. Since $\angle RTS$ and $\angle EGF$ are bisected, we know that $\frac{1}{2}m\angle RTS = \frac{1}{2}m\angle EGF$ or $m\angle RTA = m\angle EGB$. This makes $\Delta RTA \sim \Delta EGB$ by AA Similarity. Thus, since corresponding sides are proportional in similar triangles, we can state that $\frac{TA}{GB} = \frac{RT}{EG}$.

ANSWER:

Given: $\Delta RTS \sim \Delta EGF, \overline{TA}$ and $\overline{GB}$ are angle bisectors.

Prove: $\frac{TA}{GB} = \frac{RT}{EG}$



Proof: Because corresponding angles of similar triangles are congruent, $\angle R \cong \angle E$ and $\angle RTS \cong \angle EGF$. Since $\angle RTS$ and $\angle EGF$ are

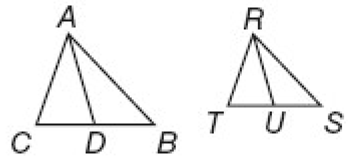
bisected, we know that $\frac{1}{2}m\angle RTS = \frac{1}{2}m\angle EGF$ or $m\angle RTA = m\angle EGB$. This makes $\angle RTA \cong \angle EGB$ and $\Delta RTA \sim \Delta EGB$ by AA Similarity. Thus, $\frac{TA}{GB} = \frac{RT}{EG}$.

19. **PROOF** Write a two-column proof of Theorem 7.10.

SOLUTION:

Theorem 7.10 states that if two triangles are similar, the lengths of corresponding medians are proportional to the lengths of corresponding sides. Since we know that $\Delta ABC \sim \Delta RST$, we can use congruent corresponding angles and proportional sides to work towards getting $\Delta ABD \sim \Delta RSU$, Since we already have $\angle B \cong \angle S$ and $\frac{AB}{RS} = \frac{CB}{TS}$, we can use the given that $\overline{AD}$ and $\overline{RU}$ are medians to prove that, $\frac{AB}{RS} = \frac{DB}{US}$ through segment addition postulate, definition of median, and substitution. Once this this established, you can prove that $\Delta ABD \sim \Delta RSU$ and, consequently, corresponding parts are proportional in these two triangles.

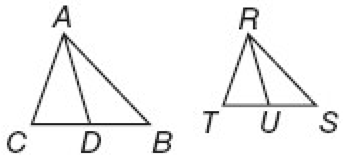
Given: $\Delta ABC \sim \Delta RST$
$\overline{AD}$ is a median of ΔABC.
$\overline{RU}$ is a median of ΔRST.
Prove: $\frac{AD}{RU} = \frac{AB}{RS}$



Proof:
Statements (Reasons)
1. $\Delta ABC \sim \Delta RST$; $\overline{AD}$ is a median of ΔABC; $\overline{RU}$ is a median of ΔRST. (Given)
2. $CD = DB$; $TU = US$ (Def. of median)
3. $\frac{AB}{RS} = \frac{CB}{TS}$ (Def. of $\sim \Delta s$)
4. $CB = CD + DB$; $TS = TU + US$ (Seg. Add. Post.)
5. $\frac{AB}{RS} = \frac{CD + DB}{TU + US}$ (Subst.)
6. $\frac{AB}{RS} = \frac{DB + DB}{US + US}$ or $\frac{2(DB)}{2(US)}$ (Subst.)
7. $\frac{AB}{RS} = \frac{DB}{US}$ (Subst.)
8. $\angle B \cong \angle S$ (Def. of $\sim$ triangles)
9. $\Delta ABD \sim \Delta RSU$ (SAS Similarity)
10. $\frac{AD}{RU} = \frac{AB}{RS}$ (Def. of $\sim \Delta s$)

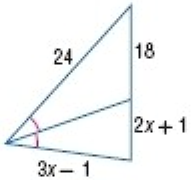
ANSWER:

Given: $\Delta ABC \sim \Delta RST$
$\overline{AD}$ is a median of ΔABC.
$\overline{RU}$ is a median of ΔRST.
Prove: $\frac{AD}{RU} = \frac{AB}{RS}$



Proof:
Statements (Reasons)
1. $\Delta ABC \sim \Delta RST$; $\overline{AD}$ is a median of ΔABC; $\overline{RU}$ is a median of ΔRST. (Given)
2. $CD = DB$; $TU = US$ (Def. of median)
3. $\frac{AB}{RS} = \frac{CB}{TS}$ (Def. of $\sim \Delta s$)
4. $CB = CD + DB$; $TS = TU + US$ (Seg. Add. Post.)
5. $\frac{AB}{RS} = \frac{CD + DB}{TU + US}$ (Subst.)
6. $\frac{AB}{RS} = \frac{DB + DB}{US + US}$ or $\frac{2(DB)}{2(US)}$ (Subst.)
7. $\frac{AB}{RS} = \frac{DB}{US}$ (Subst.)
8. $\angle B \cong \angle S$ (Def. of $\sim$ triangles)
9. $\Delta ABD \sim \Delta RSU$ (SAS Similarity)
10. $\frac{AD}{RU} = \frac{AB}{RS}$ (Def. of $\sim \Delta s$)

ALGEBRA Find *x*.

20. 

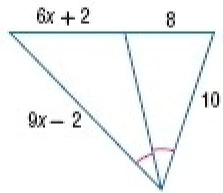
SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{18}{2x+1} = \frac{24}{3x-1}$$
$$18(3x-1) = 24(2x+1)$$
$$54x - 18 = 48x + 24$$
$$6x = 42$$
$$x = 7$$

ANSWER:

7

21. 

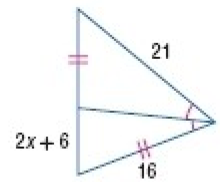
SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{6x+2}{8}=\frac{9x-2}{10}$$
$$10(6x+2)=8(9x-2)$$
$$60x+20=72x-16$$
$$12x=36$$
$$x=3$$

ANSWER:

3

22. 

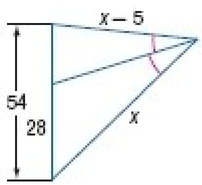
SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{2x+6}{16}=\frac{16}{21}$$
$$21(2x+6)=256$$
$$42x+126=256$$
$$42x=130$$
$$x\approx 3.1$$

ANSWER:

3.1

23. 

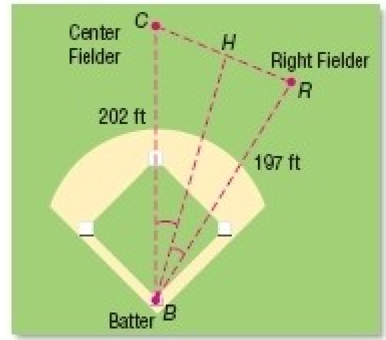
SOLUTION:

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

$$\frac{x-5}{x}=\frac{54-28}{28}$$
$$\frac{x-5}{x}=\frac{26}{28}$$
$$\frac{x-5}{x}=\frac{13}{14}$$
$$14(x-5)=13x$$
$$14x-70=13x$$
$$x=70$$

70

24. **SPORTS** Consider the triangle formed by the path between a batter, center fielder, and right fielder as shown. If the batter gets a hit that bisects the triangle at $\angle B$, is the center fielder or the right fielder closer to the ball? Explain your reasoning.



SOLUTION:

Right fielder; sample answer: According to the Triangle Bisector Theorem, an angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

proportional to the other two sides, such as $\frac{CH}{202}=\frac{RH}{197}$ or $\frac{CH}{RH}=\frac{BC}{BR}$.

Substituting, $\frac{CH}{RH}=\frac{202}{197}\approx 1.03$. Since $\frac{CH}{RH}$ is slightly greater than 1, *CH* is slightly longer than *RH*.

Therefore, the right fielder is closer to the hit.

ANSWER:

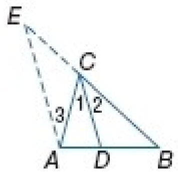
Right fielder; sample answer: Since the hit bisects the triangle, the sides opposite the right angle are proportional to the other two sides, or $\frac{CH}{RH}=\frac{BC}{BR}$. Substituting, $\frac{CH}{RH}=\frac{202}{197}\approx 1.03$. Since $\frac{CH}{RH}$ is slightly greater than 1, *CH* is slightly longer than *RH*. Therefore, the right fielder is closer to the hit.

PROOF Write a two-column proof.

25. Theorem 7.11

Given: $\overline{CD}$ bisects $\angle ACB$. By construction, $\overline{AE} \parallel \overline{CD}$.

Prove: $\frac{AD}{DB} = \frac{AC}{BC}$



SOLUTION:

Theorem 7.11 states that an angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides. Since you have a line ($\overline{CD}$) that is parallel to one side of the triangle and divides the other sides of the triangle into two parts, by the Triangle Proportionality theorem, we know that $\frac{AD}{DB} = \frac{EC}{BC}$. This looks very similar to the conclusion, except we need to replace EC with AC. Since $\overline{CD} \parallel \overline{EA}$, then $\angle E \cong \angle 2$ and $\angle 1 \cong \angle 3$. From the given information, we can also conclude that $\angle 1 \cong \angle 2$ (How?). Therefore, $\angle E \cong \angle 3$. This means that ΔECA is an isosceles triangle and $\overline{EC} \cong \overline{AC}$. Now, we can replace EC with AC in $\frac{AD}{DB} = \frac{EC}{BC}$.

Proof:

Statements (Reasons)

1. $\overline{CD}$ bisects $\angle ACB$; By construction, $\overline{AE} \parallel \overline{CD}$. (Given)
2. $\frac{AD}{DB} = \frac{EC}{BC}$ (Δ Prop. Thm.)
3. $\angle 1 \cong \angle 2$ (Def. of Angle Bisector)
4. $\angle 3 \cong \angle 1$ (Alt. Int. angle Thm.)
5. $\angle 2 \cong \angle E$ (Corr. angle Post.)
6. $\angle 3 \cong \angle E$ (Trans. Prop.)
7. $\overline{EC} \cong \overline{AC}$ (Conv. of Isos. Δ Thm.)
8. $EC = AC$ (Def. of $\cong$ segs.)
9. $\frac{AD}{DB} = \frac{AC}{BC}$ (Subst.)

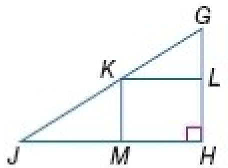
ANSWER:

Proof:

Statements (Reasons)

1. $\overline{CD}$ bisects $\angle ACB$; By construction, $\overline{AE} \parallel \overline{CD}$. (Given)
2. $\frac{AD}{DB} = \frac{EC}{BC}$ (Δ Prop. Thm.)
3. $\angle 1 \cong \angle 2$ (Def. of Angle Bisector)
4. $\angle 3 \cong \angle 1$ (Alt. Int. angle Thm.)
5. $\angle 2 \cong \angle E$ (Corr. angle Post.)
6. $\angle 3 \cong \angle E$ (Trans. Prop.)
7. $\overline{EC} \cong \overline{AC}$ (Conv. of Isos. Δ Thm.)
8. $EC = AC$ (Def. of $\cong$ segs.)
9. $\frac{AD}{DB} = \frac{AC}{BC}$ (Subst.)

26. **Given:** $\angle H$ is a right angle. L, K, and M are midpoints.
Prove: $\angle LKM$ is a right angle.



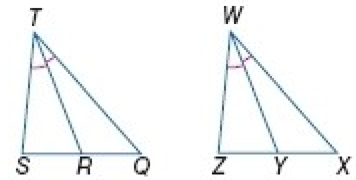
SOLUTION:
This proof is based on the idea that, if you can prove that $\angle H$, $\angle GLK$, and $\angle KLM$ are all congruent to each other and one is a right angle, then they all must be right angles. Think about how to get $\overline{JH} \parallel \overline{LK}$, $\overline{GH} \parallel \overline{KM}$ using the given information. Then, the angle relationships will follow.

Proof:
Statements (Reasons)
1. $\angle H$ is a right angle. L, K, and M are midpoints. (Given)
2. $\overline{JH} \parallel \overline{LK}$, $\overline{GH} \parallel \overline{KM}$ (Midsegment Thm.)
3. $\angle H \cong \angle GLK$ (Corr. angles Post.)
4. $\angle GLK \cong \angle LKM$ (Alt. Int. angles $\cong$)
5. $\angle GLK$ is a right angle. (Subst.)
6. $\angle LKM$ is a right angle. (Subst.)

ANSWER:
Proof:
Statements (Reasons)
1. $\angle H$ is a right angle. L, K, and M are midpoints. (Given)
$\overline{JH} \parallel \overline{LK}$, $\overline{GH} \parallel \overline{KM}$ (Midsegment Thm.)
3. $\angle H \cong \angle GLK$ (Corr. $\angle$s Post.)
4. $\angle GLK \cong \angle LKM$ (Alt. Int. $\angle$s $\cong$)
5. $\angle GLK$ is a right angle. (Subst.)
6. $\angle LKM$ is a right angle. (Subst.)

PROOF Write a two-column proof.

27. **Given:** $\Delta QTS \sim \Delta XWZ$, $\overline{TR}$ and $\overline{WY}$ are angle bisectors.
Prove: $\frac{TR}{WY} = \frac{QT}{XW}$



SOLUTION:
This proof is based on the idea that, if you can prove $\Delta QTR \sim \Delta XWY$, then you can make a proportional statement regarding corresponding sides. So, since $\Delta STQ \sim \Delta ZWX$, we already know that $\angle Q \cong \angle X$. Therefore, we just need one more pair of congruent corresponding angles. We are given that $\overline{TR}$ and $\overline{WY}$ are angle bisectors so we can state that $\angle STR \cong \angle QTR$ and $\angle ZWY \cong \angle YWX$. We also know that $\angle STQ \cong \angle ZWX$, so you can also prove that their half angles must also be congruent. Then, by AA Similarity, you can prove $\Delta QTR \sim \Delta XWY$, and consequently, $\frac{TR}{WY} = \frac{QT}{XW}$.

Proof:
Statements (Reasons)
1. $\Delta STQ \sim \Delta ZWX$, $\overline{TR}$ and $\overline{WY}$ are angle bisectors. (Given)
2. $\angle STQ \cong \angle ZWX$, $\angle Q \cong \angle X$ (Def of $\sim \Delta$ s)
3. $\angle STR \cong \angle QTR$, $\angle ZWY \cong \angle YWX$ (Def. angle bisector)
4. $m\angle STQ = m\angle STR + m\angle QTR$,
$m\angle ZWX = m\angle ZWY + m\angle XWY$ (Angle Add. Thm.)
5. $m\angle STQ = 2m\angle QTR$, $m\angle ZWX = 2m\angle XWY$ (Subst.)
6. $2m\angle QTR = 2m\angle XWY$ (Subst.)
7. $m\angle QTR = m\angle XWY$ (Div.)
8. $\angle QTR \cong \angle XWY$ (def of congruent angles)
9. $\Delta QTR \sim \Delta XWY$ (AA Similarity)

10. $\frac{TR}{WY} = \frac{QT}{XW}$ (Def of $\sim \Delta s$)

ANSWER:

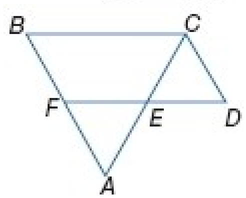
Proof:

Statements (Reasons)

1. $\Delta STQ \sim \Delta ZWX$, $\overline{TR}$ and $\overline{WY}$ are angle bisectors. (Given)
2. $\angle STQ \cong \angle ZWX$, $\angle Q \cong \angle X$ (Def of $\sim \Delta$ s)
3. $\angle STR \cong \angle QTR$, $\angle ZWY \cong \angle YWX$ (Def. angle bisector)
4. $m\angle STQ = m\angle STR + m\angle QTR$, $m\angle ZWX = m\angle ZWY + m\angle XWY$ (Angle Add. Thm.)
5. $m\angle STQ = 2m\angle QTR$, $m\angle ZWX = 2m\angle XWY$ (Subst.)
6. $2m\angle QTR = 2m\angle XWY$ (Subst.)
7. $m\angle QTR = m\angle XWY$ (Div.)
8. $\angle QTR \cong \angle XWY$ (def of congruent angles)
9. $\Delta QTR \sim \Delta XWY$ (AA Similarity)
10. $\frac{TR}{WY} = \frac{QT}{XW}$ (Def of $\sim \Delta s$)

28. **Given:** $\overline{FD} \parallel \overline{BC}$, $\overline{BF} \parallel \overline{CD}$, $\overline{AC}$ bisects $\angle C$.

Prove: $\frac{DE}{EC} = \frac{BA}{AC}$



SOLUTION:

This proof can be solved by finding two pairs of congruent corresponding angles in ΔDEC and ΔBAC, thereby proving them similar.Once they are similar, you can prove that any pair of corresponding sides are congruent. Use the given parallel lines to get Alternate Interior angles and corresponding angles congruent. Remember, from the given information, you also know that $\angle BCE \cong \angle DCE$. So put all your statements together to get two pairs of congruent angles in ΔDEC and ΔBAC.

Proof:

Statements (Reasons)

1. $\overline{FD} \parallel \overline{BC}$, $\overline{BF} \parallel \overline{CD}$, $\overline{AC}$ bisects $\angle BCD$. (Given)
2. $\angle BCE \cong \angle DCE$ (Def. $\angle$ bisector)
3. $\angle BCE \cong \angle AEF$ (Corr. $\angle$s Post.)
4. $\angle AEF \cong \angle DEC$ (Vert. $\angle$s are $\cong$)
5. $\angle BCE \cong \angle DEC$ (Trans. Prop.)
6. $\angle BAC \cong \angle DCE$ (Alt. Int $\angle$s Thm.)
7. $\Delta DEC \sim \Delta BAC$
8. $\frac{DE}{EC} = \frac{BA}{AC}$ (Def of $\sim \Delta s$)

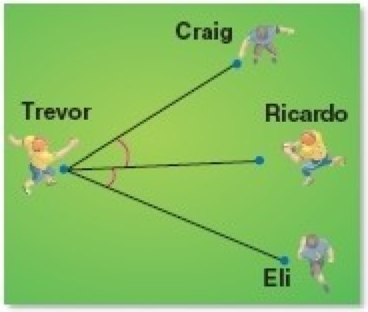
ANSWER:

Proof:

Statements (Reasons)

1. $\overline{FD} \parallel \overline{BC}$, $\overline{BF} \parallel \overline{CD}$, $\overline{AC}$ bisects $\angle BCD$. (Given)
2. $\angle BCE \cong \angle DCE$ (Def. $\angle$ bisector)
3. $\angle BCE \cong \angle AEF$ (Corr. $\angle$s Post.)
4. $\angle AEF \cong \angle DEC$ (Vert. $\angle$s are $\cong$)
5. $\angle BCE \cong \angle DEC$ (Trans. Prop.)
6. $\angle BAC \cong \angle DCE$ (Alt. Int $\angle$s Thm.)
7. $\Delta DEC \sim \Delta BAC$ (AA Similarity)
8. $\frac{DE}{EC} = \frac{BA}{AC}$ (Def of $\sim \Delta s$)

29. **SPORTS** During football practice, Trevor threw a pass to Ricardo as shown below. If Eli is farther from Trevor when he completes the pass to Ricardo and Craig and Eli move at the same speed, who will reach Ricardo to tackle him first?



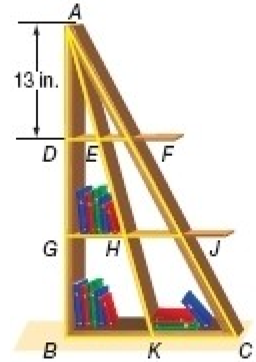
SOLUTION:

Craig; According to the Triangle Bisector Theorem, an angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides. Since the pass bisects the triangle, the sides opposite Trevor are proportional to the distances between Trevor (T) and defenders (Craig (C) and Eli (E)) , such as $\frac{CR}{CT}=\frac{RE}{ET}$. Since Craig is closer to Trevor,then he is proportionally closer to Ricardo (R).

ANSWER:

Craig

30. **SHELVING** In the triangular bookshelf shown, the distance between each of the shelves is 13 inches and $\overline{AK}$ is a median of ΔABC. If EF is $3\frac{1}{3}$ inches, what is BK?



SOLUTION:

Since $\angle ADF$ and $\angle ABC$ are congruent (assuming the shelves are parallel to each other) and $\angle DAF \cong \angle DAF$, we know $\Delta ABC \sim \Delta ADF$, by AA Similarity. Since we know that $DE = EF$ and $BK = KC$, we can form a proportion of corresponding sides:

$$\frac{AD}{DF}=\frac{AB}{BC}$$

Since AB= AD+DG+GH, DF= DE+EF, and BC=BK+KC, then we can substitute these values in the proportion as shown:

$$\frac{AD}{DE+EF}=\frac{AD+DG+GB}{BK+KC}$$

Substitute in given values and solve for BK:

$$\frac{13}{3\frac{1}{3}+3\frac{1}{3}}=\frac{13+13+13}{BK+BK}$$

$$\frac{13}{6\frac{2}{3}} = \frac{39}{2BK}$$

$$26BK = 39\left(6\frac{2}{3}\right)$$

$$26BK = 39\left(\frac{20}{3}\right)$$

$$26BK = \frac{780}{3}$$

$$BK = \frac{780}{3} \cdot \frac{1}{26}$$

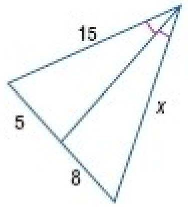
$$BK = \frac{780}{78} = 10$$

So, BK = 10 in..

ANSWER:

10 in.

31. **ERROR ANALYSIS** Chun and Traci are determining the value of x in the figure. Chun says to find x, solve the proportion $\frac{5}{8} = \frac{15}{x}$, but Traci says to find x, the proportion $\frac{5}{x} = \frac{8}{15}$ should be solved. Is either of them correct? Explain.



SOLUTION:

Chun; The Triangle Angle Bisector theorem states that an angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides. So, a correct proprotion would be $\frac{5}{15} = \frac{8}{x}$, or as Chun said $\frac{5}{8} = \frac{15}{x}$. Traci would have been correct, if the triangles were also similar to each other.

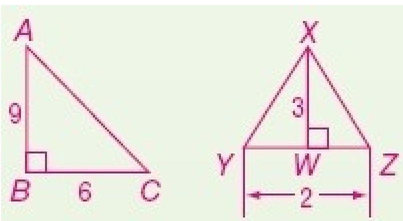
ANSWER:

Chun; by the Angle Bisector Theorem, the correct proportion is $\frac{5}{8} = \frac{15}{x}$.

32. **REASONING** Find a counterexample to the following statement. Explain.
If the measure of an altitude and side of a triangle are proportional to the corresponding altitude and corresponding side of another triangle, then the triangles are similar.

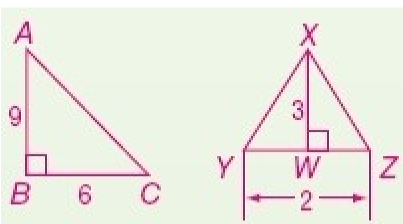
SOLUTION:

When choosing triangles that are a counterexample to this statement, choose a right triangle and a non-right triangle. In this way, they will both have an altitude but cannot be similar.



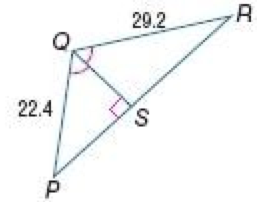
$\frac{AB}{BC}=\frac{XW}{YZ}$, but $\Delta ABC \neq \Delta XYZ$.

ANSWER:



$\frac{AB}{BC}=\frac{XW}{YZ}$, *but* $\Delta ABC \neq \Delta XYZ$.

33. **CHALLENGE** The perimeter of ΔPQR is 94 units. $\overline{QS}$ bisects $\angle PQR$. Find *PS* and *RS*.



SOLUTION:

The perimeter of triangle *PQR* = 94.

We know that *PQ* + *QR* + *PR* = 94.

Substitute given values and solve for PR:

22.4 + 29.2 + *PR* = 94
51.6 + *PR* = 94
PR = 42.4

Form a proportion using similar triangles ΔPSQ and ΔSRQ and substitute given values:

$$\frac{PS}{22.4}=\frac{SR}{29.2}$$
$$\frac{PS}{22.4}=\frac{42.4-PS}{29.2}$$
$$29.2PS=949.76-22.4PS$$
$$51.6PS=949.76$$
$$PS\approx 18.4$$

Since *PS* is about 18.4, *RS* is about 42.4 – 18.4 or 24.

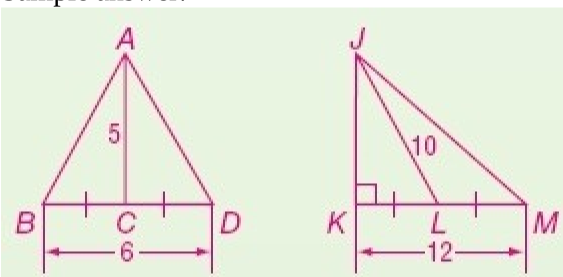
ANSWER:

PS = 18.4, *RS* = 24

34. **OPEN ENDED** Draw two triangles so that the measures of corresponding medians and a corresponding side are proportional, but the triangles are not similar.

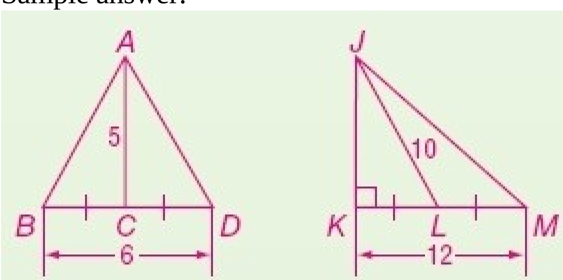
SOLUTION:

Sample answer:



ANSWER:

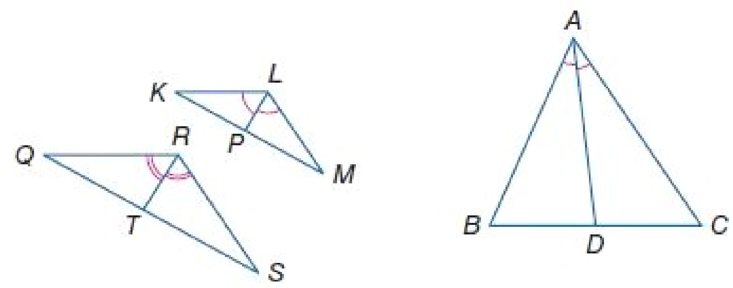
Sample answer:



35. **WRITING IN MATH** Compare and contrast Theorem 7.9 and the Triangle Angle Bisector Theorem.

SOLUTION:

Both theorems have a segment that bisects an angle and has proportionate ratios. The Triangle Angle Bisector Theorem pertains to one triangle, while Theorem 7.9 pertains to similar triangles. Unlike the Triangle Angle Bisector Theorem, which separates the opposite side into segments that have the same ratio as the other two sides, Theorem 7.9 relates the angle bisector to the measures of the sides.



$$\frac{PK}{LK} = \frac{TQ}{RQ} \qquad \frac{DA}{AB} = \frac{DC}{AC}$$

ANSWER:

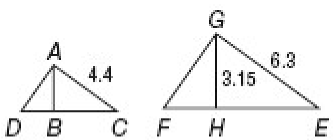
Both theorems have a segment that bisects an angle and has proportionate ratios. The Triangle Angle Bisector Theorem pertains to one triangle, while Theorem 7.9 pertains to similar triangles. Unlike the Triangle Angle Bisector Theorem, which separates the opposite side into segments that have the same ratio as the other two sides, Theorem 7.9 relates the angle bisector to the measures of the sides.

36. **ALGEBRA** Which shows 0.00234 written in scientific notation?
A 2.34×10^5
B 2.34×10^3
C 2.34×10^{-2}
D 2.34×10^{-3}

SOLUTION:
To write 0.00234 in scientific notation, we have to move the decimal point three places to the right, and compensate by multiplying by 10^{-3}. So, the correct answer is D.

ANSWER:
D

37. **SHORT RESPONSE** In the figures below, $\overline{AB} \perp \overline{DC}$ and $\overline{GH} \perp \overline{FE}$.



If $\triangle ACD \sim \triangle GEF$, find AB.

SOLUTION:
If two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides.

$$\frac{AB}{GH} = \frac{AC}{GE}$$
$$\frac{AB}{3.15} = \frac{4.4}{6.3}$$
$$AB = 2.2$$

ANSWER:
2.2

38. Quadrilateral *HJKL* is a parallelogram. If the diagonals are perpendicular, which statement must be true?
F Quadrilateral *HJKL* is a square.
G Quadrilateral *HJKL* is a rectangle.
H Quadrilateral *HJKL* is a rhombus.
J Quadrilateral *HJKL* is an isosceles trapezoid.

SOLUTION:
If the diagonals of a parallelogram are perpendicular, then it must be a

ANSWER:
H

39. **SAT/ACT** The sum of three numbers is 180. Two of the numbers are the same, and each of them is one third of the greatest number. What is the least number?
A 15
B 30
C 36
D 45
E 60

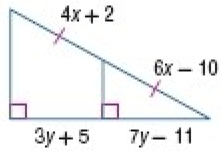
SOLUTION:
Let *x* be the least number.
Form an equation for the given situation.

$$\begin{aligned} x + x + 3x &= 180 \\ 5x &= 180 \\ x &= 36 \end{aligned}$$

So, the least number is C.
The correct choice is C.

ANSWER:
C

ALGEBRA Find x and y.



SOLUTION:

We can prove that the small right triangle and the big right triangle are similar because of AA Similarity.

We are given that $4x + 2 = 6x - 10$. Solve for x:

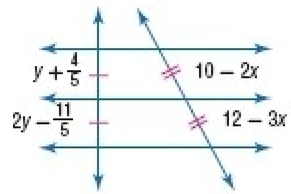
$$\begin{aligned} 4x + 2 &= 6x - 10 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

By Corollary 7.2, $7y - 11 = 3y + 5$. Solve for y.

$$\begin{aligned} 7y - 11 &= 3y + 5 \\ 4y &= 16 \\ y &= 4 \end{aligned}$$

ANSWER:

x=6; y=4

41. 

SOLUTION:

We are given that $10 - 2x = 12 - 3x$ and $y + \frac{4}{5} = 2y - \frac{11}{5}$.

Solve for x.

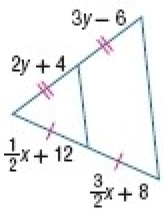
$$\begin{aligned} 10 - 2x &= 12 - 3x \\ -2 &= -x \\ x &= 2 \end{aligned}$$

Solve for y.

$$\begin{aligned} y + \frac{4}{5} &= 2y - \frac{11}{5} \\ -y &= -\frac{15}{5} \\ y &= 3 \end{aligned}$$

ANSWER:

$x = 2; y = 3$

42. 

SOLUTION:

We are given that $\frac{1}{2}x+12=\frac{3}{2}x+8$ and $2y+4=3y-6$.

Solve for x.

$$\frac{1}{2}x+12=\frac{3}{2}x+8$$
$$\frac{1}{2}x-\frac{3}{2}x=8-12$$
$$-x=-4$$
$$x=4$$

Solve for y.

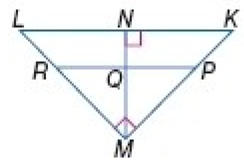
$$2y+4=3y-6$$
$$-y=-10$$
$$y=10$$

ANSWER:

$x = 4$; $y = 10$

Find the indicated measure(s).

43. If $\overline{PR}\parallel\overline{KL}$, $KN = 9$, $LN = 16$, $PM = 2(KP)$, find KP, KM, MR, ML, MN, and PR.



SOLUTION:

We can prove that $\Delta LMK \sim \Delta MNK$, by AA Similarity.

Also, since PM = 2(KP), then KM= KP +PM= KP+2(KP)=3(KP).

Write a proportion and solve for KP:

$$\frac{MK}{LK}=\frac{NK}{MK}$$
$$\frac{3(KP)}{25}=\frac{9}{3(KP)}$$
$$9(KP)^2=225$$
$$(KP)^2=25$$
$$KP=5$$

So, KP=5 and KM=3(5)=15.

To find RP, we can use the similar triangles $\Delta LMK \sim \Delta RMP$ (AA Similarity) to write the proportion:

$$\frac{MK}{LK}=\frac{PM}{RP}$$
$$\frac{15}{25}=\frac{10}{RP}$$
$$15RP=250$$
$$RP=16\frac{2}{3}$$

Use the Pythagorean Theorem to find LM:

$$LM^2 + MK^2 = LK^2$$
$$LM^2 + 15^2 = 25^2$$
$$LM^2 + 225 = 625$$
$$LM^2 = 400$$
$$LM = 20$$

Finally, we can set up a proportion using similar triangles $\Delta LMK \sim \Delta MNK$ to find MN:

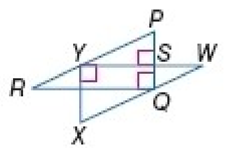
$$\frac{LM}{MK} = \frac{MK}{NK}$$
$$\frac{20}{15} = \frac{MK}{9}$$
$$180 = 15MK$$
$$12 = MN$$

So, KP = 5, *KM* = 15, *MR* = $13\frac{1}{3}$, *ML* = 20, *MN* = 12, *PR* = $16\frac{2}{3}$

ANSWER:

KP = 5, *KM* = 15, *MR* = $13\frac{1}{3}$, *ML* = 20, *MN* = 12, *PR* = $16\frac{2}{3}$

44. If $\overline{PR} \parallel \overline{WX}$, *WX* = 10, *XY* = 6, *WY* = 8, *RY* = 5, and *PS* = 3, find *PY*, *SY*, and *PQ*.



SOLUTION:

Since $\angle PSY$, $\angle PQR$, *and* $\angle WYX$ are right angles, then we know that they are congruent. Also, $\angle PYS \cong \angle WYX$ (Alternate interior angles theorem) and $\overline{YW} \parallel \overline{RQ}$ (Converse of corresponding angles theorem). Therefore, we can use similar triangles and parallel lines to write proportional statements:

Since $\Delta WYX \sim \Delta YSP$ (AA Similarity), we can write proportions to find SY and PY:

$$\frac{WY}{YX} = \frac{SY}{SP} \qquad \frac{WX}{YX} = \frac{PY}{SP}$$
$$\frac{8}{6} = \frac{SY}{3} \qquad \frac{10}{6} = \frac{PY}{3}$$
$$24 = 6SY \qquad 30 = 6PY$$
$$4 = SY \qquad 5 = PY$$

Since $\overline{YW} \parallel \overline{RQ}$, then we can write a proportion concerning the sides of ΔPRQ, using the Triangle Proportion Theorem:

$$\frac{PY}{YR} = \frac{PS}{SQ}$$
$$\frac{5}{5} = \frac{3}{SQ}$$
$$15 = 5SQ$$
$$3 = SQ$$

Since PQ= PS+SQ, then PQ = 3+3=6 units.

Therefore, *PY* = 5, *SY* = 4, *PQ* = 6

ANSWER:

PY = 5, *SY* = 4, *PQ* = 6

45. **GEESE** A flock of geese flies in formation. Prove that

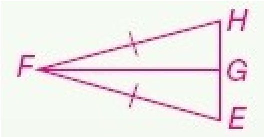
$\Delta EFG \cong \Delta HFG$ if $\overline{EF} \cong \overline{HF}$ and that G is the midpoint of $\overline{EH}$.
Refer to Page 502.

SOLUTION:

To prove that triangles are congruent, you must use one of the congruent triangle postulates or theorems, like SAS, AAS, ASA, SSS, or HL. Since G is the midpoint of $\overline{EH}$, we know that makes $\overline{EG} \cong \overline{GH}$. To determine which one to use for this proof, mark the given relationships on your figure.

Given: $\overline{EF} \cong \overline{HF}$

G is the midpoint of $\overline{EH}$.
Prove: $\boldsymbol{\Delta EFG \cong \Delta HFG}$
Proof:



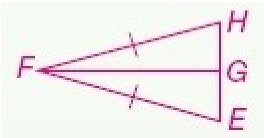
<u>Statements (Reasons)</u>

1. $\overline{EF} \cong \overline{HF}$; G is the midpoint of $\overline{EH}$. (Given)
2. $\overline{EG} \cong \overline{GH}$ (Def. of midpoint)
3. $\overline{FG} \cong \overline{FG}$ (Reflexive Prop.)
4. $\Delta EFG \cong \Delta HFG$ (SSS)

ANSWER:

Given: $\overline{EF} \cong \overline{HF}$

G is the midpoint of $\overline{EH}$.
Prove: $\boldsymbol{\Delta EFG \cong \Delta HFG}$
Proof:



<u>Statements (Reasons)</u>

1. $\overline{EF} \cong \overline{HF}$; G is the midpoint of $\overline{EH}$. (Given)
2. $\overline{EG} \cong \overline{GH}$ (Def. of midpoint)
3. $\overline{FG} \cong \overline{FG}$ (Reflexive Prop.)
4. $\Delta EFG \cong \Delta HFG$ (SSS)

Find the distance between each pair of points.

46. $E(-3, -2)$, $F(5, 8)$

SOLUTION:

Use the distance formula.

$$\begin{aligned} EF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5-(-3))^2 + (8-(-2))^2} \\ &= \sqrt{(8)^2 + (10)^2} \\ &= \sqrt{64+100} \\ &= \sqrt{164} \\ &\approx 12.8 \end{aligned}$$

ANSWER:

$\sqrt{164} \approx 12.8$

47. $A(2, 3)$, $B(5, 7)$

SOLUTION:

Use the distance formula.

$$\begin{aligned} AB &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(5-2)^2+(7-3)^2} \\ &= \sqrt{(3)^2+(4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

ANSWER:

5

48. $C(-2, 0)$, $D(6, 4)$

SOLUTION:

Use the distance formula.

$$\begin{aligned} CD &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(6-(-2))^2+(4-0)^2} \\ &= \sqrt{(8)^2+(4)^2} \\ &= \sqrt{64+16} \\ &= \sqrt{80} \\ &\approx 8.9 \end{aligned}$$

ANSWER:

$\sqrt{80} \approx 8.9$

49. $W(7, 3)$, $Z(-4, -1)$

SOLUTION:

Use the distance formula.

$$\begin{aligned} WZ &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-4-7)^2+(-1-3)^2} \\ &= \sqrt{(-11)^2+(-4)^2} \\ &= \sqrt{121+16} \\ &= \sqrt{137} \\ &\approx 11.7 \end{aligned}$$

ANSWER:

$\sqrt{137} \approx 11.7$

50. $J(-4, -5)$, $K(2, 9)$

SOLUTION:

Use the distance formula.

$$\begin{aligned} JK &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(2-(-4))^2+(9-(-5))^2} \\ &= \sqrt{(6)^2+(14)^2} \\ &= \sqrt{36+196} \\ &= \sqrt{232} \\ &\approx 15.2 \end{aligned}$$

ANSWER:

$\sqrt{232} \approx 15.2$

51. $R(-6, 10)$, $S(8, -2)$

SOLUTION:

Use the distance formula.

$$\begin{aligned} RS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8-(-6))^2 + (-2-10)^2} \\ &= \sqrt{(14)^2 + (-12)^2} \\ &= \sqrt{196 + 144} \\ &= \sqrt{340} \\ &\approx 18.4 \end{aligned}$$

ANSWER:

$\sqrt{340} \approx 18.4$